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Remarks on the amplitude of the decay

$$\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$$

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Abstract

We point out some properties of the amplitude of the dipion transition $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ in relation to the recently reported results of a CLEO analysis of form factors in this amplitude. We find that the reported significant complex phase between two of the form factors under the assumption that the third form factor is zero, is not consistent with the picture where the phase shifts arise due to the final state interaction in the $\pi\pi$ channel. It is also shown that in an analysis that uses no information on the polarization of both the initial and the final Υ resonances it is impossible in principle to determine all the relevant terms. We suggest that a study of a simple correlation between the direction of the total momentum of the two pions and the axis of the initial beams is sufficient to resolve the ambiguity in the fit for the form factors.

The transitions between states of heavy quarkonium with emission of two pions present a case study in both the internal works of heavy quarkonia and the low-energy dynamics of light mesons in QCD. The general constraints on the amplitudes of such transitions with soft pions follow from the chiral algebra[1, 2]. The specifics of heavy quarkonium is revealed in that any such transition can be viewed, in a way, as a two-stage process: the quarkonium transition produces gluon field, which in turn creates the light mesons. Considering quarkonium as a compact object in the scale of typical energy in the transition, one can apply the multipole expansion in QCD for interaction of the heavy quark system with the glue field[3], while the creation of the two pions by this field can be described with the help of low-energy theorems in QCD[4, 5]. It is well known that some of the observed $\pi\pi$ transitions in charmonium, namely, $\psi(2S) \rightarrow J/\psi \pi\pi$, $\Upsilon(2S) \rightarrow \Upsilon(1S) \pi\pi$, and $\Upsilon(3S) \rightarrow \Upsilon(2S) \pi\pi$ display the behavior, in particular the spectrum of the invariant mass of the dipion, that is expected under the simplest assumptions about the corresponding heavy quarkonium multipoles. On the other hand, the transition $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ is long known to defy such straightforward predictions by displaying a peculiar double-peaked spectrum of $m_{\pi\pi}$ ¹.

Given such behavior, a further understanding of the anomalous behavior in the transition $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ would likely greatly benefit from an input from experiment providing more details about the structure of the decay amplitude. A study of this decay based on a large data sample has been recently reported by CLEO[8]. In this analysis they use the parametrization of the amplitude[1] motivated by the soft pion limit:

$$\begin{aligned} \mathcal{M} = & \left[A (q^2 - 2m_\pi^2) + \lambda m_\pi^2 \right] (\epsilon_1 \cdot \epsilon_2) + B E_1 E_2 (\epsilon_1 \cdot \epsilon_2) \\ & + C [(p_1 \cdot \epsilon_1)(p_2 \cdot \epsilon_2) + (p_2 \cdot \epsilon_1)(p_1 \cdot \epsilon_2)] , \end{aligned} \quad (1)$$

where ϵ_1^μ and ϵ_2^μ are the polarization amplitudes of the initial and the final vector resonances, p_1 and p_2 are the 4-momenta of the two pions, E_1 and E_2 are their energies in the rest frame of the initial state, and $q = p_1 + p_2$ is the total 4-momentum of the dipion, so that $q^2 = m_{\pi\pi}^2$. Finally, A , B , C , and λ are the form factors, which should be considered constant in the soft pion limit, but are generally functions of kinematic variables beyond this limit. The term of order m_π^2 proportional to λ is the ‘ σ term’[1] and it is fixed at zero in the fits of Ref.[8].

¹It is interesting to note that the data on the recently observed two-pion transitions from $\Upsilon(4S)$ indicate that the transition $\Upsilon(4S) \rightarrow \Upsilon(1S) \pi\pi$ has a well-behaved $m_{\pi\pi}$ spectrum[6, 7], while the spectrum in the decay $\Upsilon(4S) \rightarrow \Upsilon(2S) \pi\pi$ possibly resembles that in $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ [7]. It is however unclear at present to what extent effects of $B\bar{B}$ meson pairs can contribute to hadronic transitions from $\Upsilon(4S)$.

The spectrum of $m_{\pi\pi}$ and the angular distribution in the angle θ_X between the direction of motion of the two pions in their center of mass frame and the direction of the vector \vec{q} are then used to fit the values of the ratios $b = B/A$ and $c = C/A$. The results of this analysis suggest that the ratio b has a significant imaginary part: if c is fixed as $c = 0$, the fit for b yields[8] $\text{Re}(b) = -2.523 \pm 0.031$ and $\text{Im}(b) = \pm 1.189 \pm 0.051$ (obviously, the sign of the imaginary part cannot be determined in such fit). If c is allowed to float, it is found that $|c| < 1.09$ at 90% C.L., and a certain correlation between $|b|$ and $|c|$ is noted in the fit.

The purpose of our present paper is to elaborate on several points related to the description of the decay amplitude and on the possibilities of studying the details of such description from data. Specifically, we point out that using only the phase-space distribution over $m_{\pi\pi}$ and θ_X and no information on polarization of either the initial or final Υ resonance, it is impossible in principle to unambiguously determine the ratios b and c , unless c is exactly equal to zero. In other words, if b and c are allowed to be complex, there is a continuous set of transformations of b and c that does not change the distribution of the decay rate over $m_{\pi\pi}$ and θ_X . This degeneracy becomes a discrete two-fold ambiguity if b and c are constrained to be real.

Furthermore, the imaginary part of the amplitude is related by the unitarity condition to rescattering of the decay products. Given an apparent absence of a strong rescattering in the exotic channel with the quantum numbers of $\Upsilon\pi$, any complex phase behavior in the considered amplitude can arise only from $\pi\pi$ rescattering, which is reduced to the relative phase between the S and D wave $\pi\pi$ scattering phases in the isoscalar $I = 0$ state. The amplitudes for production of these partial waves combine in b with coefficients having different dependence on $m_{\pi\pi}$. For this reason any complex phase of b cannot be approximated as constant. Moreover, the data[8] suggest that the production of the D wave is very small, so that the interference between the S and D waves possible in parts of the phase space (but not e.g. in the $m_{\pi\pi}$ spectrum) cannot be significant, and effectively the relevant ratio b is essentially real. Thus we believe that the result[8] claiming a sizable complex phase of b is very likely to be modified by further analyses.

Finally, a better way of independently determining the coefficients b and c , which does not suffer from the mentioned ambiguity, should use at least some basic information about the polarization of either the initial or the final Υ resonances. We point out that for the e^+e^- annihilation setting the apparently simplest correlation sensitive to the polarization-dependent amplitude is that between the direction of the initial beams and the direction of

the motion of the dipion, i.e. of the vector \vec{q} .

For consideration of the effects of different terms of the amplitude in the observable phase space distribution and also for evaluating the significance of the $\pi\pi$ rescattering it is helpful to write the decay amplitude as a sum of partial waves[9]:

$$\mathcal{M} = S (\epsilon_1 \cdot \epsilon_2) + D_1 \ell_{\mu\nu} \frac{P^\mu P^\nu}{P^2} (\epsilon_1 \cdot \epsilon_2) + D_2 q_\mu q_\nu \epsilon^{\mu\nu} + D_3 \ell_{\mu\nu} \epsilon^{\mu\nu} . \quad (2)$$

In this expression P is the 4-momentum of the initial resonance. The tensor $\ell_{\mu\nu}$ corresponds to a D -wave spatial tensor made out of momenta of the pions in their *c.m.* frame. Namely, using the notation $r = p_1 - p_2$, this tensor is defined as[5]

$$\ell_{\mu\nu} = r_\mu r_\nu + \frac{1}{3} \left(1 - \frac{4m_\pi^2}{q^2} \right) (q^2 g_{\mu\nu} - q_\mu q_\nu) . \quad (3)$$

Finally, $\epsilon^{\mu\nu}$ stands for the spin-2 tensor made from the polarization amplitudes of the resonances

$$\epsilon^{\mu\nu} = \epsilon_1^\mu \epsilon_2^\nu + \epsilon_1^\nu \epsilon_2^\mu + \frac{2}{3} (\epsilon_1 \cdot \epsilon_2) \left(\frac{P^\mu P^\nu}{P^2} - g_{\mu\nu} \right) . \quad (4)$$

The terms in the expression (2) describe an S wave and three possible types of D -wave motion: the term with D_1 corresponds to a D wave in the c.m. system of the two pions correlated with the overall motion of the dipion in the rest frame of the initial state, the D_2 term describes the D -wave motion of the dipion as a whole, correlated with the spins of the Υ resonances, and finally, the D_3 term corresponds to the correlation between the spins and the D -wave motion in the c.m. frame of the dipion. One can also notice that the S and D_1 terms contain an overall spin-0 combination of the quarkonium polarizations, so that there is no interference between these two terms and those with D_2 and D_3 , if no polarization information in the rate is used. In particular, the distribution of the rate studied in Ref.[8] can be written as

$$\frac{d\Gamma}{d\cos\theta_X dq} \propto \overline{|\mathcal{M}|_X^2} \sqrt{q_0^2 - q^2} \sqrt{q^2 - 4m_\pi^2} , \quad (5)$$

where $\overline{|\mathcal{M}|_X^2}$ stands for the square of the amplitude appropriately averaged/summed over all the variables except for θ_X and q^2 ,

$$\begin{aligned} \overline{|\mathcal{M}|_X^2} &= |S|^2 - \frac{2}{3} (1 - 3\cos^2\theta_X) (q_0^2 - q^2) \left(1 - \frac{4m_\pi^2}{q^2} \right) \text{Re}(SD_1^*) \\ &+ \frac{1}{9} (1 - 3\cos^2\theta_X)^2 (q_0^2 - q^2)^2 \left(1 - \frac{4m_\pi^2}{q^2} \right)^2 |D_1|^2 + \frac{8}{9} (q_0^2 - q^2)^2 |D_2|^2 \end{aligned} \quad (6)$$

$$\begin{aligned}
& - \frac{8}{27} (1 - 3 \cos^2 \theta_X) (q^2 + 2q_0^2) (q_0^2 - q^2) \left(1 - \frac{4m_\pi^2}{q^2}\right) \text{Re}(D_2 D_3^*) \\
& + \frac{8}{9} (q^2 - 4m_\pi^2)^2 \left[1 + \frac{1}{3} (1 + 3 \cos^2 \theta_X) \frac{q_0^2 - q^2}{q^2} + (1 - 3 \cos^2 \theta_X)^2 \frac{(q_0^2 - q^2)^2}{9(q^2)^2}\right] |D_3|^2,
\end{aligned}$$

where q_0 stands for the total energy of the two pions in the rest frame of the initial Υ resonance: $q_0 = (P \cdot q)/\sqrt{P^2} = (M'^2 - M^2 + q^2)/2M' \approx M' - M$ with M' (M) standing for the mass of the initial (final) Υ resonance.

The four form factors in Eq.(2) can be expressed in terms of the form factors in the expression (1):

$$\begin{aligned}
S &= \left(A + \frac{1}{3}C\right) (q^2 - 2m_\pi^2) + \lambda m_\pi^2 + \frac{1}{12} \left(B - \frac{2}{3}C\right) \left[3q_0^2 - (q_0^2 - q^2) \left(1 - \frac{4m_\pi^2}{q^2}\right)\right] \\
D_1 &= -\frac{1}{4} \left(B - \frac{2}{3}C\right), \quad D_2 = \frac{1}{6}C \left(1 + \frac{2m_\pi^2}{q^2}\right), \quad D_3 = -\frac{1}{4}C.
\end{aligned} \tag{7}$$

It can be noted that the D_2 and D_3 terms in Eq.(7) are both proportional to C , thus if λ is set at zero and no polarization information is being used, the shape of the distribution of the decay rate over the phase space is determined only by the parameters

$$\left|\frac{C}{A + C/3}\right| = \left|\frac{c}{1 + c/3}\right| \quad \text{and} \quad \frac{B - 2C/3}{A + C/3} = \frac{b - 2c/3}{1 + c/3}. \tag{8}$$

However, if b and c are allowed to be complex numbers, there is a continuous set of transformations of b and c that leave the quantities (8) intact:

$$c \rightarrow \tilde{c} = \frac{3c e^{i\phi}}{3 + c(1 - e^{i\phi})} \quad \text{and} \quad b \rightarrow \tilde{b} = \frac{3b - 2c(1 - e^{i\phi})}{3 + c(1 - e^{i\phi})}. \tag{9}$$

In case both b and c are constrained to be real, there still remains a two-fold ambiguity, corresponding to setting $e^{i\phi} = -1$:

$$c \rightarrow \tilde{c} = -\frac{3c}{3 + 2c} \quad \text{and} \quad b \rightarrow \tilde{b} = \frac{3b - 4c}{3 + 2c}. \tag{10}$$

The noticed ambiguity can be resolved if a polarization information could be included in the analysis. This would also provide a more direct access to fitting the spin-dependent form factor C . We illustrate this conclusion by writing the distribution of the decay rate over the angle θ_q between the axis of the initial e^+ and e^- beams and the total spatial momentum \vec{q} of the dipion:

$$\frac{d\Gamma}{d\cos\theta_q dq} \propto |\overline{\mathcal{M}}|_q^2 \sqrt{q_0^2 - q^2} \sqrt{q^2 - 4m_\pi^2}, \tag{11}$$

with $\overline{|\mathcal{M}|_q^2}$ now standing for the square of the amplitude averaged/summed over all the variables except for θ_q and q^2 ,

$$\begin{aligned}
\overline{|\mathcal{M}|_q^2} = & |S|^2 - \frac{2}{3} (1 - 3 \cos^2 \theta_q) (q_0^2 - q^2) \operatorname{Re}(SD_2^*) + \frac{10}{9} \left(1 - \frac{3}{5} \cos^2 \theta_q\right) (q_0^2 - q^2)^2 |D_2|^2 \\
& + \frac{4}{45} (q_0^2 - q^2)^2 \left(1 - \frac{4m_\pi^2}{q^2}\right)^2 |D_1|^2 \\
& - \frac{4}{135} (1 - 3 \cos^2 \theta_q) (q^2 + 2q_0^2) (q_0^2 - q^2) \left(1 - \frac{4m_\pi^2}{q^2}\right)^2 \operatorname{Re}(D_1 D_3^*) \\
& + \frac{8}{9} (q^2 - 4m_\pi^2)^2 \left[1 + \frac{47}{60} \left(1 - \frac{21}{47} \cos^2 \theta_q\right) \frac{q_0^2 - q^2}{q^2} + \left(1 - \frac{3}{5} \cos^2 \theta_q\right) \frac{(q_0^2 - q^2)^2}{9(q^2)^2}\right] |D_3|^2 .
\end{aligned} \tag{12}$$

One can notice that there is only interference in the angular distribution between the S and D_2 terms as well as between D_1 and D_3 , since the former two terms correspond to the S -wave motion of the pions in their c.m. frame, while the latter two describe the D -wave motion. Clearly, the presence of the SD_2 interference term in the angular distribution should facilitate an observation of the C form factor (proportional to D_2), even if it is somewhat small in comparison with A and B .

The rescattering between the two pions gives rise to phases of the terms in the amplitude. These phases however depend only on q^2 and on the angular momentum of the pions in their c.m. frame. Thus the terms S and D_2 receive the S -wave scattering phase factor $\exp i\delta_0$ and the terms D_1 and D_3 get the D -wave factor $\exp i\delta_2$. The $\pi\pi$ scattering phases have been a subject of many studies (see e.g in Ref.[10]). In the context of the present discussion it should be clearly understood that including these phases implies going beyond the leading soft pion limit, which is likely necessary anyway for considering the transition $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi\pi$ with a relatively high energy release. In such situation a description of the transition amplitude in terms of the partial waves, as in Eq.(2), is more consistent than using the form factors defined by the parametrization in Eq.(1). Indeed, according to Eq.(7) the terms D_2 and D_3 are both proportional to the form factor C in Eq.(1). The $\pi\pi$ rescattering however modifies in a different way the terms D_2 and D_3 , so that it would be impossible to describe them in terms of one and the same form factor C . In other words, the parametrization in Eq.(1) is only suitable in the low energy limit. Going beyond this limit requires a more general parametrization of the amplitude. On the other hand the expression in Eq.(2) with the coefficients being functions of q^2 is a general expression for the S -wave and all possible types of D -wave motion, and is limited only by the absence of higher partial waves.

It can be mentioned in connection with the discussion of the complex phases, that the distribution described by the equations (11) and (12) is in fact not sensitive to the phase shifts due to the $\pi\pi$ rescattering, so that the interfering terms in Eq.(12) are relatively real. Therefore an observation of a deviation from this behavior would signal either a presence of higher partial waves, or complex phase effects from rescattering in the exotic $\Upsilon\pi$ channel. An explanation of the peculiar spectrum in the decay $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ based on four-quark states has been suggested[11, 12, 13] in the past, but has never had a phenomenological success. It would thus be most interesting if a nontrivial dynamics in this channel manifested itself in such a subtle effect.

Finally, a remark is due on the relative significance of the quarkonium spin dependent part of the amplitude, described by the form factor C in Eq.(1) or by D_2 and D_3 in Eq.(2). This term is naturally suppressed by the inverse of the heavy quark mass, and normally should be small in comparison with the dominant S term. This appears to be indeed the case for the ‘well behaved’ dipion transitions. However it is not necessarily the case for the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ decay. Indeed, one may argue that the overall rate of this transition is suppressed, so that in fact the S term is small and does not dominate over the spin-dependent part[9]. Clearly, a definite input from experiment would be extremely helpful in exploring this possibility. As discussed previously in this paper, a study of the polarization-correlated angular distributions, e.g. of the one described by Eqs. (11) and (12), would provide a more direct access to the spin dependent terms.

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